1. Suppose we have a collection of 15 marbles in a container. 40 % of the marbles are red and 60 % of the marbles are black. The orange marbles weigh 6.00 grams and the black marbles weigh 8.00 grams.

   a) Calculate the average mass of the marbles in the container. (clearly show how you arrived at your answer.)

   \[
   \text{# of red marbles} = 15 \times \left( \frac{40 \text{ red marbles}}{100 \text{ marbles}} \right) = 6 \text{ red marbles}
   \]

   \[
   \text{# of black marbles} = 15 \times \left( \frac{60 \text{ black marbles}}{100 \text{ marbles}} \right) = 9 \text{ black marbles}
   \]

   \[
   \frac{6.00 + 6.00 + 6.00 + 6.00 + 6.00 + 8.00 + 8.00 + 8.00 + 8.00 + 8.00 + 8.00 + 8.00 + 8.00 + 8.00 + 8.00 \text{ grams}}{15 \text{ marbles}} = 7.2 \text{ grams}
   \]

   or

   \[
   \text{average mass} = \frac{6 \text{ RM} \times 6.00 \text{ grams} + 9 \text{ BM} \times 8.00 \text{ grams}}{15 \text{ marbles}}
   \]

   \[
   = \frac{108.0 \text{ grams}}{15 \text{ marbles}} = 7.20 \text{ grams}
   \]

   b) Do any marbles in the container have the same mass as the average mass?

   No the masses of the two marbles is 6.00 g (red) or 8.00 grams (black). There are no marbles with a mass of 7.20 grams.

2. Suppose we have another collection of 40 marbles in a different container. 40 % of the marbles are red and 60 % of the marbles are black. The orange marbles weigh 6.00 grams and the black marbles weigh 8.00 grams.

   a) Calculate the average mass of the marbles in the container.

   \[
   \text{average mass} = \frac{40 \times 0.40 \times 6.00 \text{ grams} + 40 \times 0.60 \times 8.00 \text{ grams}}{40 \text{ marbles}}
   \]

   \[
   = \frac{288. \text{ grams}}{40 \text{ marbles}} = 7.20 \text{ grams}
   \]

3. Outline your strategy for calculating the average mass of a collection of orange and black marbles if the total number of marbles is not know.

We can re-write the equation used in Q1 and Q2 using the variable $T_M$ for the total number of marbles in the sample; $FA$ for the fractional abundance for each marble, $MM$ for the marble mass.

\[
\text{average mass} = \frac{T_M \cdot FA_{\text{red}} \cdot MM_{\text{red}} + T_M \cdot FA_{\text{black}} \cdot MM_{\text{black}}}{T_M}
\]

\[
\text{average mass} = \frac{T_M \cdot 0.40 \cdot 6.00 \text{ grams} + T_M \cdot 0.60 \cdot 8.00 \text{ grams}}{T_M}
\]

factoring the numerator, we have

\[
\text{average mass} = T_M \cdot \frac{0.25 \cdot 5.00 \text{ grams} + 0.75 \cdot 7.00 \text{ grams}}{T_M}
\]

the total marbles cancels and the equation reduces to,

\[
\text{average mass} = 0.20 \cdot 6.00 \text{ grams} + 0.60 \cdot 8.00 \text{ grams}
\]

So we see that the total number of marbles cancels out of the equation.

\[
\text{average mass} = \text{fraction of marbles}_1 \cdot \text{mass of marble}_1 + \text{fraction of marbles}_2 \cdot \text{mass of marble}_2
\]

4. Suppose we have a collection of marbles in a container. 20% of the marbles are orange and 80% of the marbles are white. The orange marbles weigh 4.00 grams and the white marbles weigh 10.00 grams.

a) Calculate the average mass of the marbles in the container.

We can re-write the equation used in Q3 using the variable $T_M$ for the total number of marbles in the sample.

\[
\text{average mass} = \frac{T_M \cdot 0.20 \cdot 4.00 \text{ grams} + T_M \cdot 0.80 \cdot 10.00 \text{ grams}}{T_M}
\]

factoring the numerator, we have

\[
\text{average mass} = T_M \cdot \frac{0.20 \cdot 4.00 \text{ grams} + 0.80 \cdot 10.00 \text{ grams}}{T_M}
\]

the total marbles cancels and the equation reduces to,

\[
\text{average mass} = 0.20 \cdot 4.00 \text{ grams} + 0.80 \cdot 10.00 \text{ grams} = 8.80 \text{ grams}
\]

So we see that the total number of marbles cancels out of the equation.

\[
\text{average mass} = \text{fraction of marbles}_1 \cdot \text{mass of marble}_1 + \text{fraction of marbles}_2 \cdot \text{mass of marble}_2
\]
5. The element boron is composed of two different isotopes, $^{10}\text{B}$ and $^{11}\text{B}$. The percent abundance of $^{10}\text{B}$ is 19.78% and the percent abundance of $^{11}\text{B}$ is 80.22%. The relative atomic mass of $^{10}\text{B}$ is 10.01294 u and the relative atomic mass of $^{11}\text{B}$ is 11.00931 u. Calculate the (relative weighted) average atomic mass of boron.

The equation we derived in Q3 can now be expressed in terms of isotopic mass and fractional abundance of the isotope.

$$\text{average mass} = \sum (\text{fraction abundance}_i \times \text{isotopic mass}_i)$$

$$\text{average mass} = \text{fraction abundance}_1 \times \text{isotopic mass}_1 + \text{fraction abundance}_2 \times \text{isotopic mass}_2$$

Substituting the data from this problem,

$$\text{average mass} = 0.1978 \times 10.01294 \text{ u} + 0.8022 \times 11.00931 \text{ u} = 10.81 \text{ u}$$

6. If you could reach in and pick a single atom from a sample of boron what would be the most probable mass of the atom of boron you selected. Explain.

The most probable mass of a boron atom is 11.00931 u since that isotope has the highest percentage abundance.